First Math

Partial Differentiation

आवश्यक सूत्र partial differentiation

$$z = \tan^{-1}\left(\frac{y}{x}\right)$$

$$\frac{\partial z}{\partial x} = \frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot \frac{-y}{x^2} \qquad \frac{\partial^2 z}{\partial y^2}$$

$$= \frac{1}{1+\frac{y^2}{x^2}} \cdot \frac{-y}{x^2}$$

$$= \frac{1}{\frac{x^2+y^2}{x^2}} \cdot \frac{-y}{x^2}$$

$$= \frac{x^2}{x^2+y^2} \cdot \frac{-y}{x^2}$$

$$= \frac{-y}{x^2+y^2}$$

$$\frac{\partial z}{\partial x} = \frac{-y}{x^2 + y^2}$$

$$\frac{\partial^2 z}{\partial x \, \partial y}$$

$$\frac{\partial^2 z}{\partial y^2} = \frac{(x^2 + y^2) \cdot 0 - (-y)(2x + 0)}{(x^2 + y^2)^2}$$

$$\frac{\partial^2 z}{\partial x \, \partial y} = \frac{(x^2 + y^2)(-1) - (-y)(0 + 2y)}{(x^2 + y^2)^2}$$

$$= \frac{y(2x)}{(x^2 + y^2)^2}$$

$$= \frac{-x^2 - y^2 + 2y^2}{(x^2 + y^2)^2}$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{2xy}{(x^2 + y^2)^2}$$

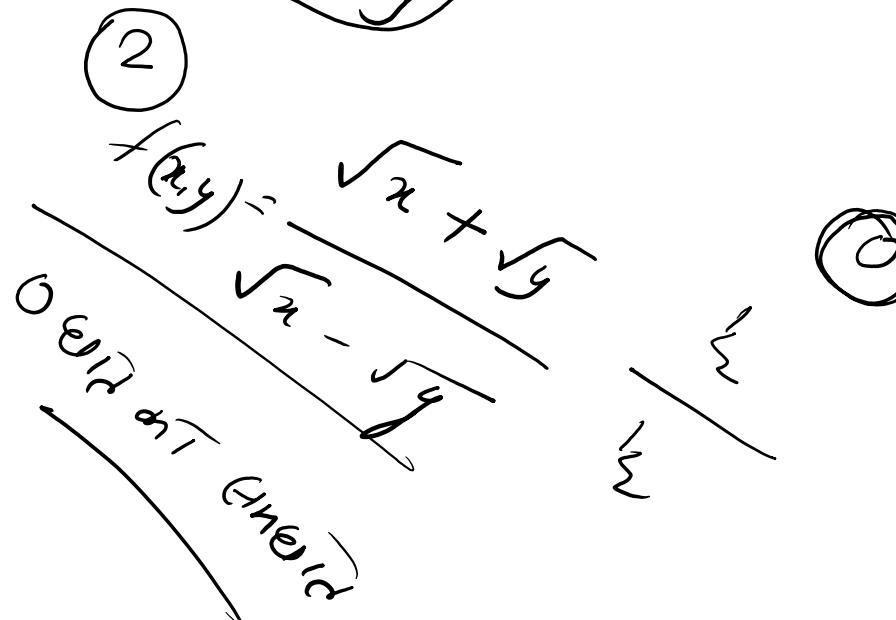$$= \frac{y^2 - x^2}{(x^2 + y^2)^2}$$

Homogeneous function समघात फलन

$$f(x, y) = x^3 + 3x^2y + 3xy^2 + y^3$$

3 घात का समघात फलन है।

$$f(x, y) = \frac{x^3 - y^3}{x + y} \qquad \frac{3}{1}$$

2 घात का समघात फलन है।

$$\boxed{\frac{x^2 - y^3}{x + y}}$$

②

$$f(x, y) = \frac{\sqrt{x} + \sqrt{y}}{\sqrt{x} - \sqrt{y}} \qquad \frac{\frac{1}{2}}{\frac{1}{2}}$$

0 घात का समघात

⓪

$$f(x,y) = \sin^{-1}\left(\frac{y}{x}\right)$$

शून्य घाते को समघात फलन हैं

$$f(x,y) = x\,\sin^{-1}\left(\frac{y}{x}\right)$$

1 घाते को समघात फलन हैं।

$$f(x,y) = xy\,\sin^{-1}\left(\frac{y}{x}\right)$$

2 घाते को समघात फलन हैं।

$$u = x\,\sin^{-1}\frac{x}{y}$$

সমাধান করঃ

$$f(x,y) = a_0 x^n + a_1 x^{n-1} y^1 + a_2 x^{n-2} y^2 + \text{------} + a_n x^0 y^n$$

$$= x^n \left[a_0 + a_1 \frac{y}{x} + a_2 \frac{y^2}{x^2} + \text{-----} a_n \frac{y^n}{x^n} \right] \underset{x^2}{\textcircled{x}}$$

$$= x^n \left[a_0 + a_1 \frac{y}{x} + a_2 \left(\frac{y}{x}\right)^2 + \text{-----} + a_n \left(\frac{y}{x}\right)^n \right]$$

$$= x^n F\left(\frac{y}{x}\right)$$

$$\boxed{f(x,y) = x^n F\left(\frac{y}{x}\right)}$$

n ঘাত বিশিষ্ট সমঘাত ফাংশন

समघात फलनों पर आयलर प्रमेय $\Rightarrow$

$f(x,y)$ समघात फलन f की घात $= n$

$$\boxed{x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf}$$

$$= 3f$$

$$= 3\left(x^3 + 3x^2y + 3xy^2 + y^3\right)$$

$f(x,y) = \underbrace{(x^3 + 3x^2y + 3xy^2 + y^3)}_{3 \text{ घात}}$
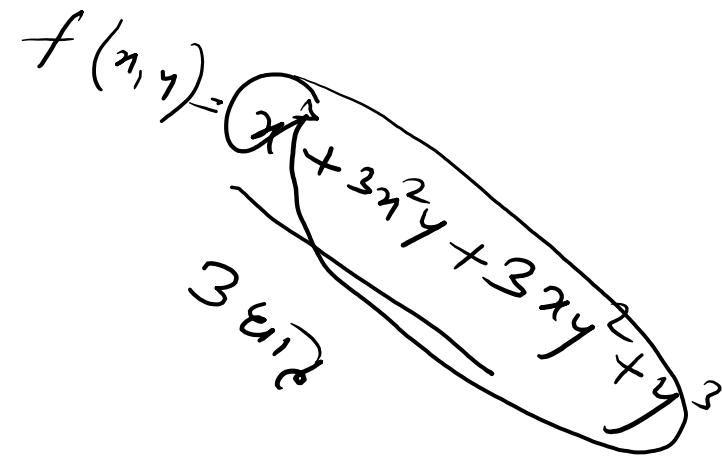
$$f(x,y,z) = 3x^2yz + 5xy^2z + 4z^4$$

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} + z\frac{\partial f}{\partial z} = ?$$

f. ঘাতি কা সমঘাতি ফলন।

∴ আমরা জানি যে

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} + z\frac{\partial f}{\partial z} = nf$$

$$= 4f$$

$$= 4(3x^2yz + 5xy^2z + 4z^4)$$